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Exam. Code : 211001
Subject Code : 3837

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M.Sc. Mathematics 1st Semester

MECHANICS—I

Paper—MATH-554

Time Allowed—Three Hours] [Maximum Marks—100

Note :— Attempt **FIVE** questions in all, selecting at least **ONE** question from each Section. All questions carry equal marks.

SECTION—A

1. Obtain the radial and transverse components of acceleration of a particle which describe the plane curve $r = f(\theta)$ in the form :

$$\ddot{r} - r\dot{\theta}^2, \frac{1}{r}\frac{d}{d\theta}(r^2\dot{\theta}).$$

If the curve is the equiangular spiral $r = a \exp(\theta \cot \alpha)$ and if the radius vector to the particle has a constant angular velocity, show that the resultant acceleration of the particle makes an angle 2α with the radius vector and its magnitude $\frac{v^2}{r}$, where v is the speed of the particle.

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2. (a) Show that two equal and opposite rotations of a rigid body about distinct parallel axes are equivalent to a translation of the body.

(b) A rigid lamina is moving in its own plane and one point A in it has velocity $\vec{u}$ relative to a fixed origin 0. If $\vec{w}$ is the angular velocity of the lamina, show that the point P in it has velocity $\vec{u} + \vec{w} \times \vec{r}$ relative to 0, where $\vec{r} = \overrightarrow{AP}$. Hence or otherwise prove that the position vector $\vec{r}\,'$ of the instantaneous centre I of the lamina relative to A is given by $\vec{r}\,' = (\vec{w} \times \vec{u})/w^2$.

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SECTION—B

3. (a) Explain the Principle of conservation of energy for a single particle.

(b) Define the impulse of a force over a finite time interval and derive the equation of impulsive motion of a particle. Also show that the K.E. gained is $\frac{\vec{I}}{2}(\vec{v}_1 + \vec{v}_2)$ where an impulse $\vec{I}$ changes the velocity of a particle of mass m from $\vec{v}_1$ to $\vec{v}_2$.

4. (a) Show that the acceleration of a particle P moving along a plane curve c is $\ddot{s}\hat{t} + (\dot{s}^2/\rho)\hat{n}$, where s denotes the arc length along c, $\hat{t}, \hat{n}$ are unit vectors along the tangent and normal at P respectively and ρ is the radius of curvature at P.

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(b) Show that the rate of increase of angular momentum about the axis is equal to the moment of the resultant force which acts on the particle.

SECTION—C

5. (a) Derive an expression for the differential equation of a particle moving in a central orbit in pedal co-ordinates.

(b) If $P = \mu(u^2 - au^3)$, where $a > 0$ and a particle is projected from an apse at a distance a from the centre of force with a velocity $(\mu c/a^2)^{1/2}$, where $a > c$, prove that the other apsidal distance of the orbit is $a(a + c)/(a - c)$ and find the apsidal angle.

6. (a) A fixed nucleus S, having positive charge, Ze repels a particle P having mass m and positive charge e'. P is projected from a great distance at infinity with initial speed v_0 in a direction whose perpendicular distance from S is d, the medium being a vacuum. Show that its ultimate direction of motion makes an angle ϕ with an initial direction where $\cot\frac{\phi}{2}$ (dm v_0^2/Zee').

(b) A comet travelling in an elliptic orbit round the sun under an attraction μ/r^2 per unit mass has its tangential velocity increased a small amount δv. Taking 2a to be the major axis and e the eccentricity of the former orbit, show that the comet's least distance from the sun is increased by

$$4\,4\delta v[a^3(1-e)/\mu(1+e)]^{1/2}.$$

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SECTION—D

7. (a) State and prove the parallel axes theorem for moments of inertia and for products of inertia for a system of particles.

   (b) A square of side 2a has particles of masses m, 2 m, 3 m, 4 m at its vertices. Find the principal moments of inertia at the centre of the square and also the directions of the principal axes.

8. (a) Define equimomental systems. Show that a solid cuboid of mass M is equimomental with masses $\frac{M}{24}$ at the mid points of its edges and $\frac{M}{2}$ at its centre.

   (b) In coplanar distribution, show that the M.I. attains extreme values along the principal axes through O in the plane of distribution.

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